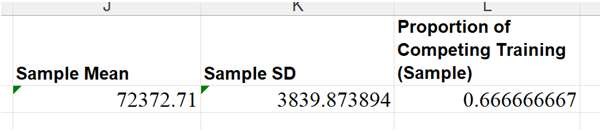


Sampling Distributions

Sampling Distribution of *x*

Sampling Distribution of *p*

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Sampling Distributions

* Chose a random sample of 30 managers and calculated
  + Sample Mean = 𝑥
  + Sample Proportion = 𝑝
* 𝑥 is the point estimator of the

population mean, 𝜇

* + Point Estimate: 𝑥 = $72,372.71
* 𝑝 is the point estimator of the

population proportion, 𝑝

* + Point Estimate: 𝑝 = 0.67

**What would happen if we chose a different sample of 30 managers?\***

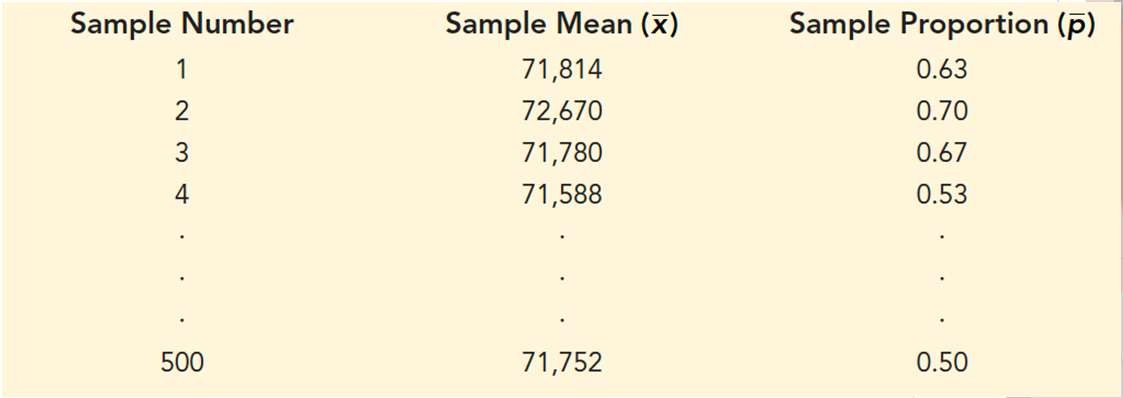
19



Sampling Distributions

If we picked 30 random managers 500 times, the results might look something like this:

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Sampling Distributions

Make a frequency and relative frequency distribution of the 𝑥 results:

|  |  |  |
| --- | --- | --- |
| **Mean Annual Salary ($)** | **Frequency** | **Relative Frequency** |
| 69,500.00–69,999.99 | 2 | 0.004 |
| 70,000.00–70,499.99 | 16 | 0.032 |
| 70,500.00–70,999.99 | 52 | 0.104 |
| 71,000.00–71,499.99 | 101 | 0.202 |
| 71,500.00–71,999.99 | 133 | 0.266 |
| 72,000.00–72,499.99 | 110 | 0.220 |
| 72,500.00–72,999.99 | 54 | 0.108 |
| 73,000.00–73,499.99 | 26 | 0.052 |
| 73,500.00–73,999.99 | 6 | 0.012 |
| **Totals:** | **500** | **1.000** |

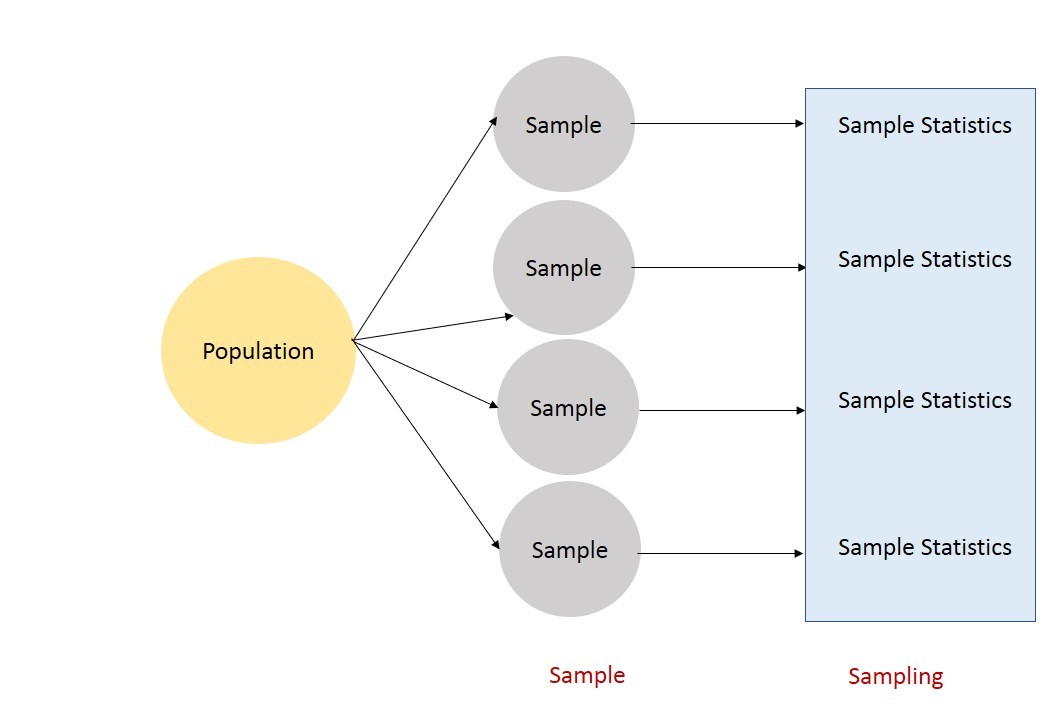
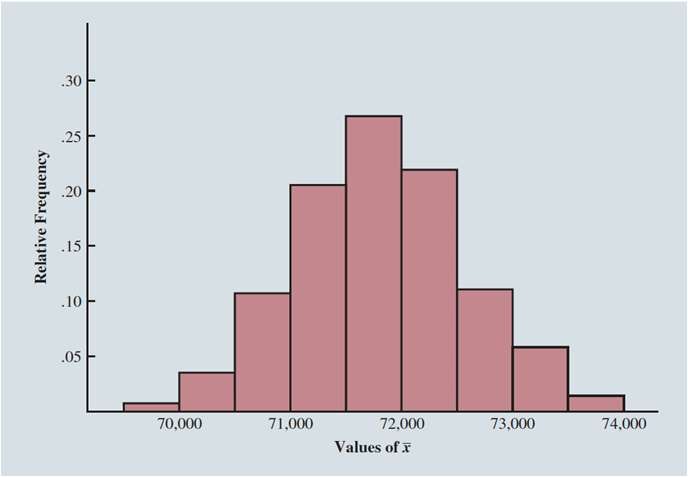
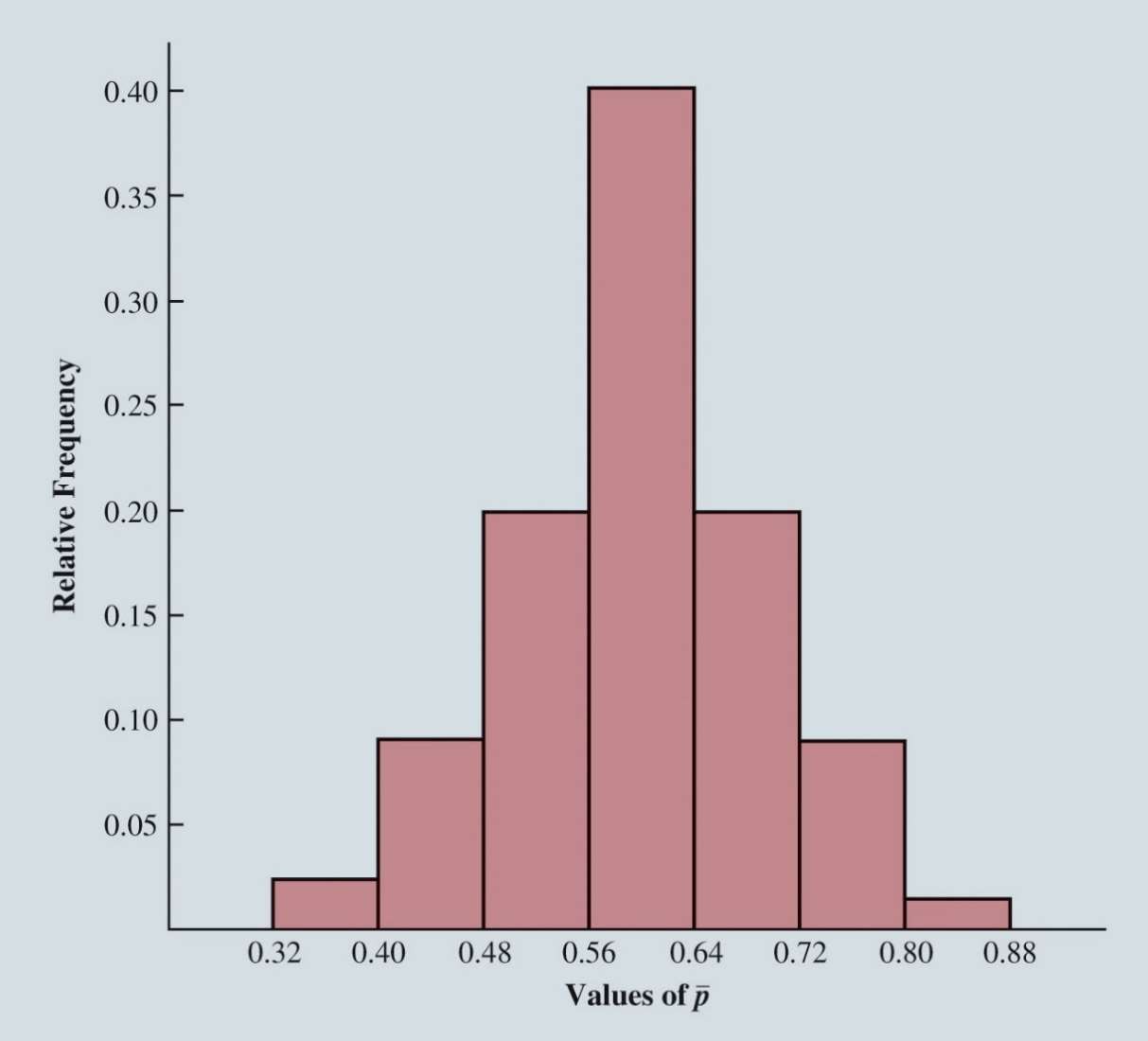
21



Sampling Distributions

Use the frequency distribution to make a histogram:

22



Sampling Distributions

* 𝑥̅ and 𝑝̅ are random variables so they have:
  + A Mean, or Expected Value
  + A Standard Deviation
  + Probability Distribution or **Sampling Distribution**
    - Each Probability Distribution has a **characteristic shape** or **form**
    - Knowing the **Sampling Distribution** of 𝑥̅, and its properties helps us gauge how close the sample mean, 𝑥 is to the true Population Mean, 𝜇

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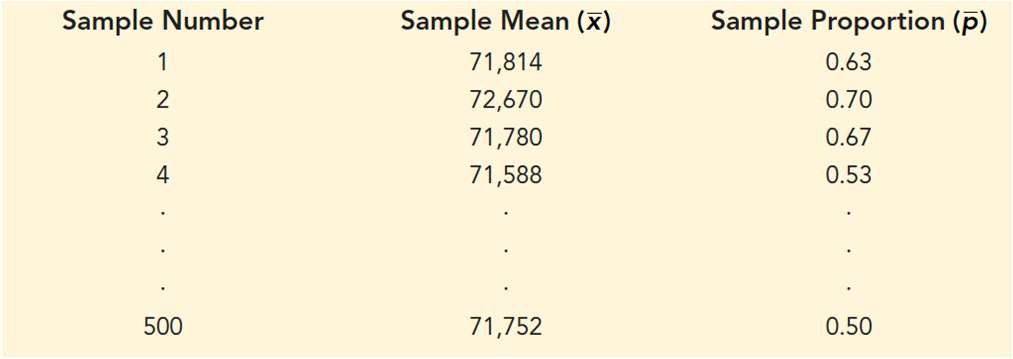
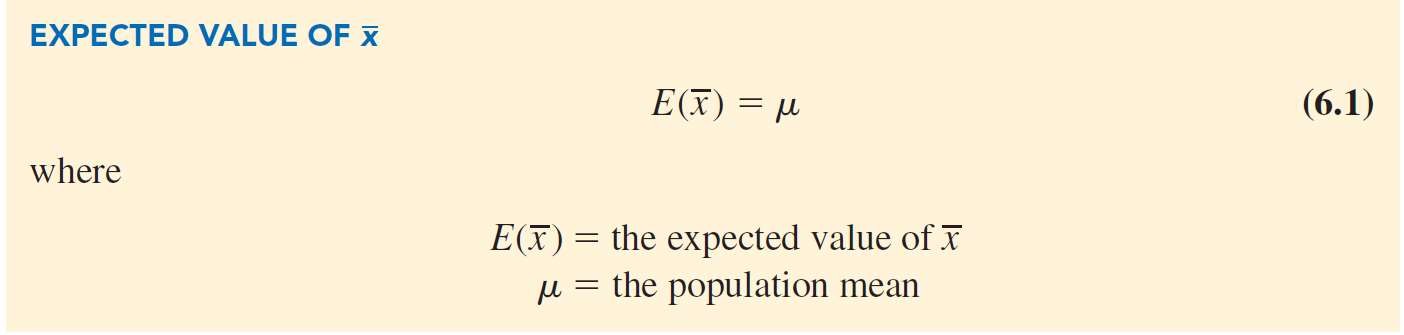


Sampling Distributions

* The Expected Value of the sample mean 𝑥
  + Is the mean average of all possible values of 𝑥 that can be generated by the various simple random samples
* Turns out:
  + The average of all the 𝑥's we could get by sampling 30 managers over and over actually equals the Population Mean, 𝜇

**What is it called when the expected value of a point estimator equals the population parameter?\***

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Sampling Distributions

71,814 + 72, 670 + 71,780 + 71,588 + … + 71,752

500 = $71,800 = 𝜇 = 𝑃𝑜𝑝𝑢𝑙𝑎𝑡𝑖𝑜𝑛 𝑀𝑒𝑎𝑛

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Sampling Distributions

The formula for the standard deviation of x depends on whether the population is finite or infinite.

Using the following notation:

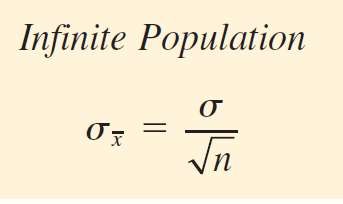
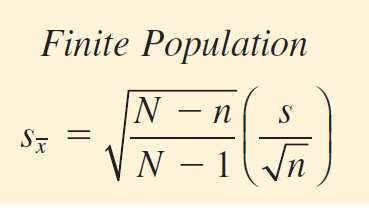
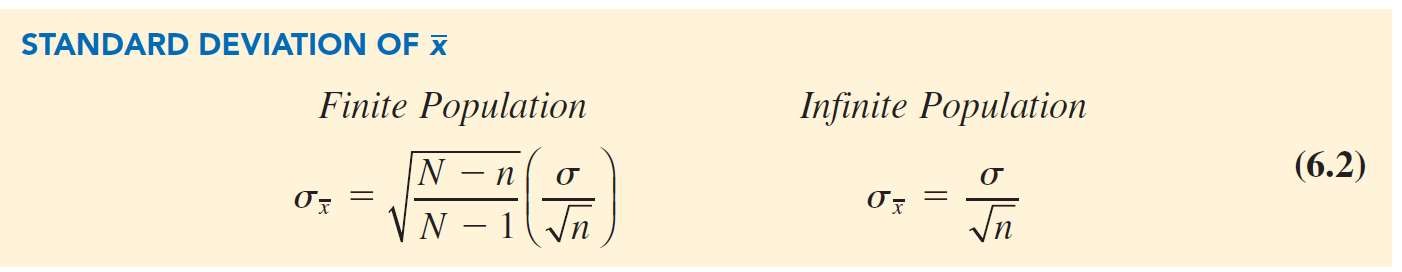
𝜎x = the standard deviation of 𝑥, or the standard error of the mean.

𝜎 = the standard deviation of the population.

𝑛 = the sample size.

𝑁 = the population size.

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Sampling Distributions

* **Finite population correction factor**:

*N*  *n N*  1

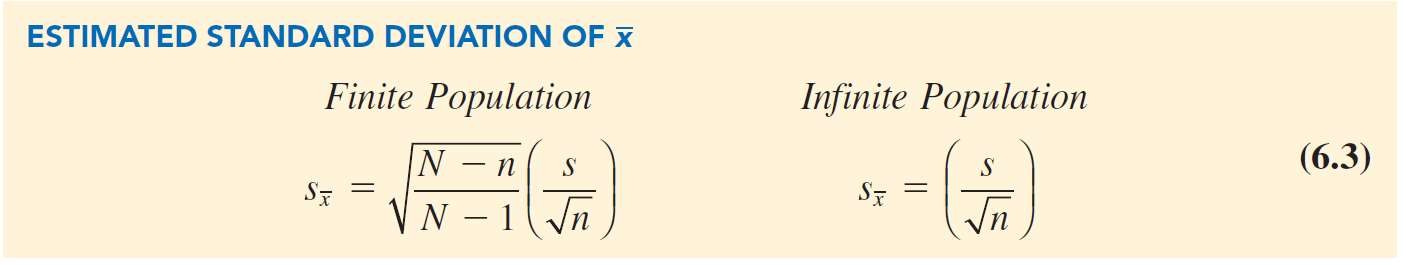
* In many practical sampling situations,
  + The finite population correction factor is close to 1
  + So, the difference between the finite and infinite standard deviations is negligible.

In general, you can use

𝑛

when, 𝑁 < 0.05

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Sampling Distributions

Estimated standard error: *s* = = 3,348 = 611.3.

*s*

*x*

*n*

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True standard error: ** = = = 730.3.

** 4,000

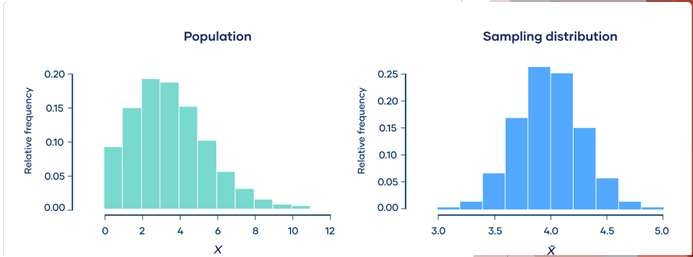
*x*

*n*

30

The difference between *sx* and *x* is due to sampling error.

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Sampling Distributions

* When the population has a normal distribution,
  + The sampling distribution of 𝑥 is normally distributed for any sample size
* When the population does not have a normal distribution
  + The central limit theorem is helpful in identifying the shape of the sampling distribution of 𝑥

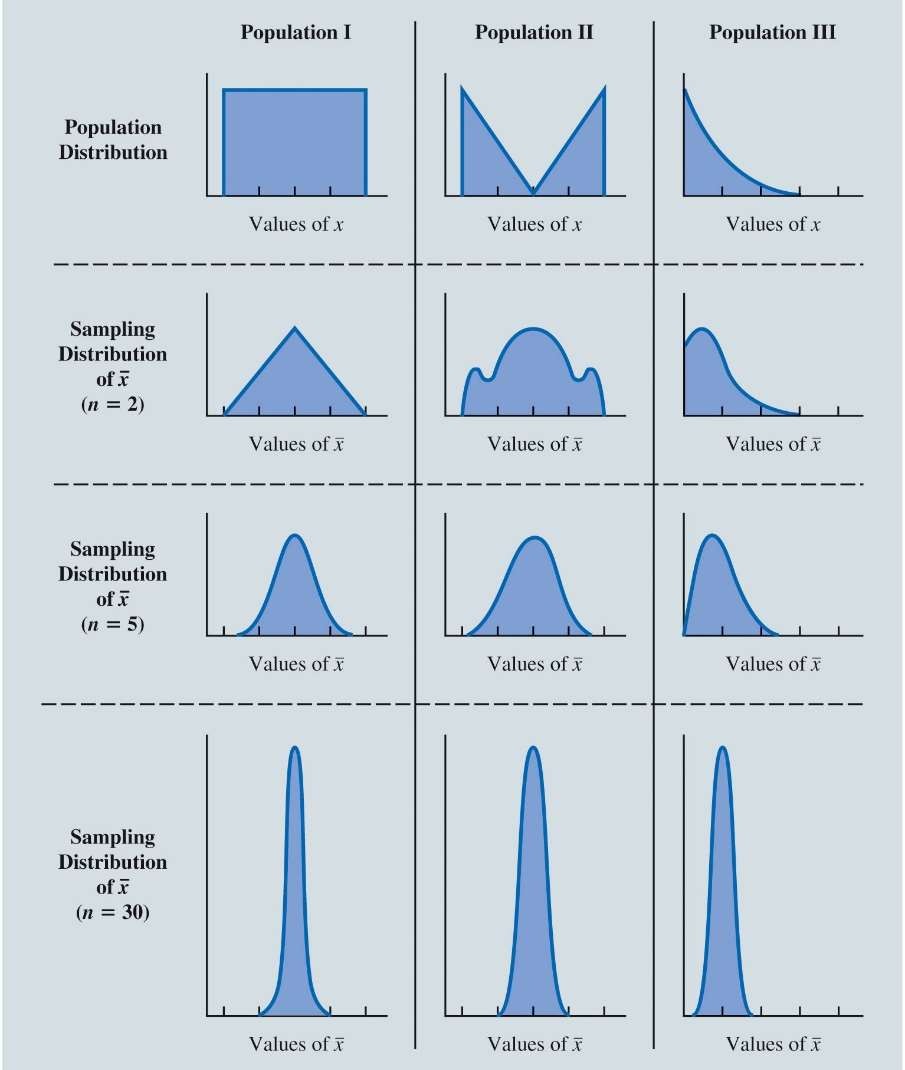
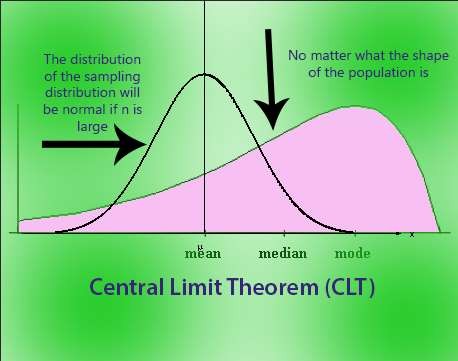
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Sampling Distributions

* **Central Limit Theorem**
  + Distribution of the sample mean 𝑥 can be approximated by a normal distribution as the sample size becomes large.

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Sampling Distributions

Central Limit Theorem for Three Populations

* Top panel shows that none of the populations are normally distributed.
* Bottom three panels show the shape of the sampling distribution for samples *n* = 2, *n* = 5, and *n*

= 30.

* For sample size of 30 or more, it looks closer to normal
* We can assume then, for a sample of 30 or more, the sampling distribution can be approximated by normal distribution.

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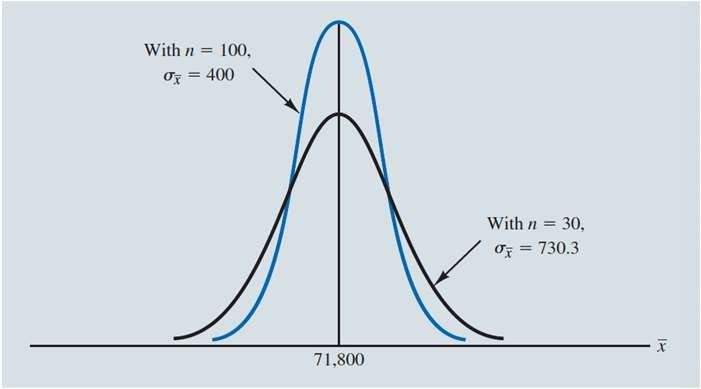
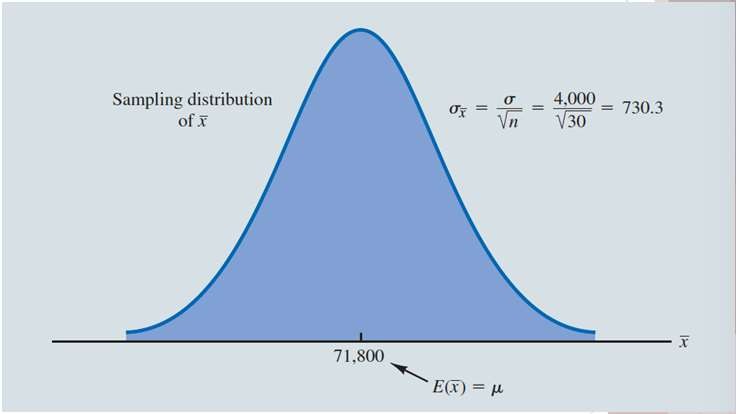


Sampling Distributions

Sampling Distribution of 𝑥 for the Mean Annual Salary of a Simple Random Sample of 30 EAI

Employees

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Sampling Distributions

A Comparison of the Sampling Distributions of 𝑥 for Simple Random Samples of:

𝑛 = 30 and 𝑛 = 100 EAI Employees.

33



Sampling Distributions

Sampling Distribution of *p*:

The sample proportion *p* is the point estimator of the population proportion *p*.

The formula for computing the sample proportion is:

*p* = *x*

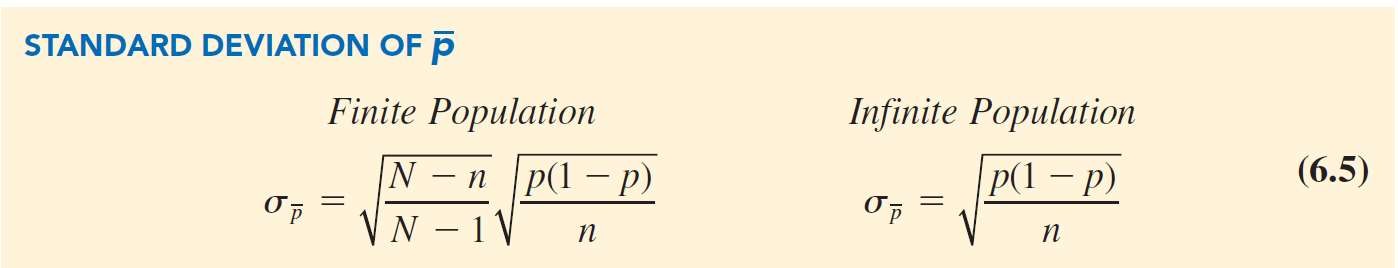
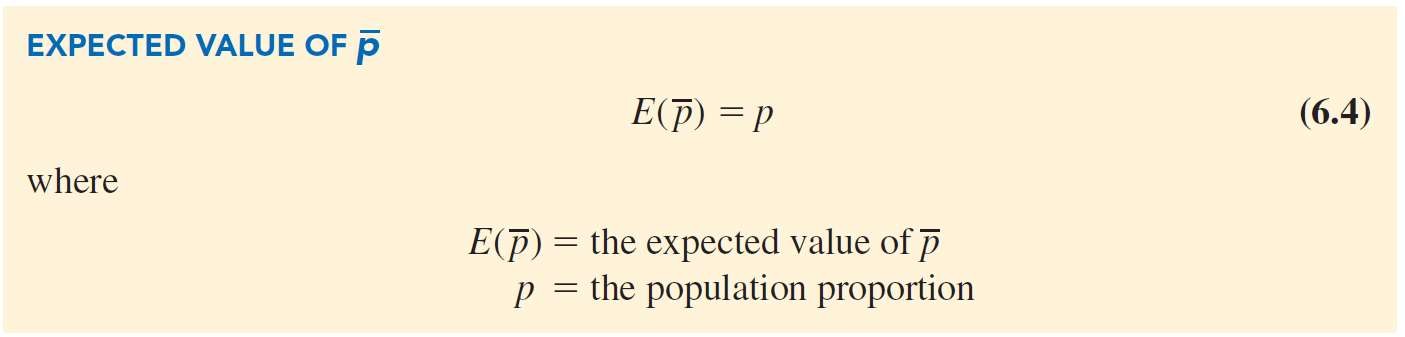
*n*

where

*x* = the number of elements in the sample that possess the characteristic of interest.

*n* = sample size.

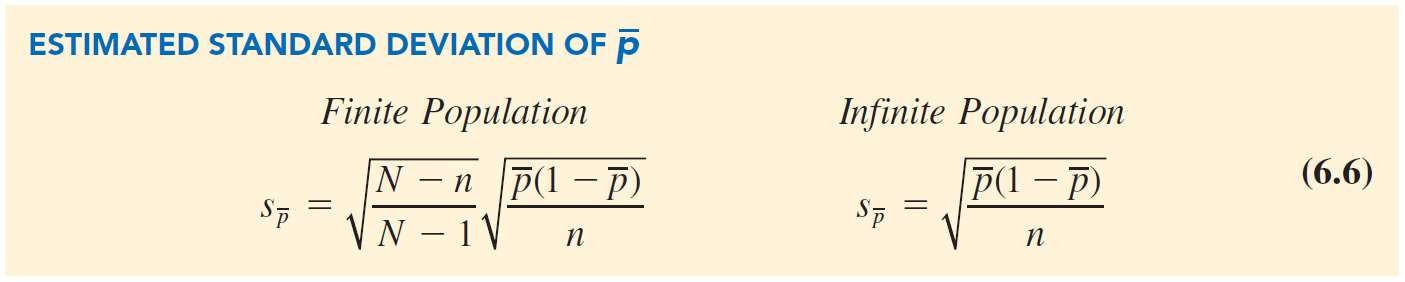
34



Sampling Distributions

Sampling distribution of *p*: The sampling distribution of *p* is the probability distribution of all possible values of the sample proportion *p*.

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Sampling Distributions

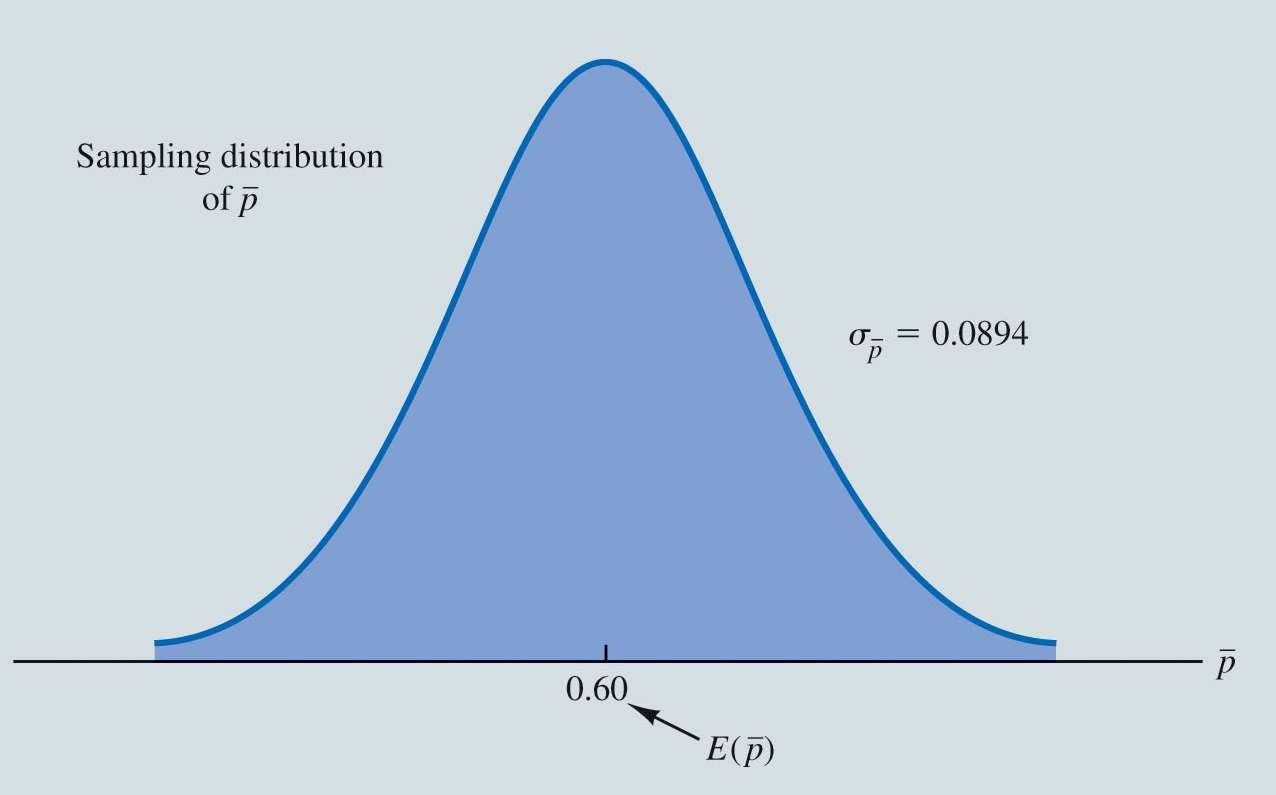
distribution whenever *np*  5 and *n*(1  *p*)  5.

For our example: 𝑝 = 0.63, n = 30

The sampling distribution of *p* can be approximated by a normal

𝑛𝑝 = 30 0.63 = 18.9 > 5 𝑎𝑛𝑑 𝑛(1 − 𝑝) = 30 0.37¯ = 11.1 > 5

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Sampling Distributions

Sampling Distribution of 𝑝 for the Proportion of EAI Employees Who Participated in the Management Training Program

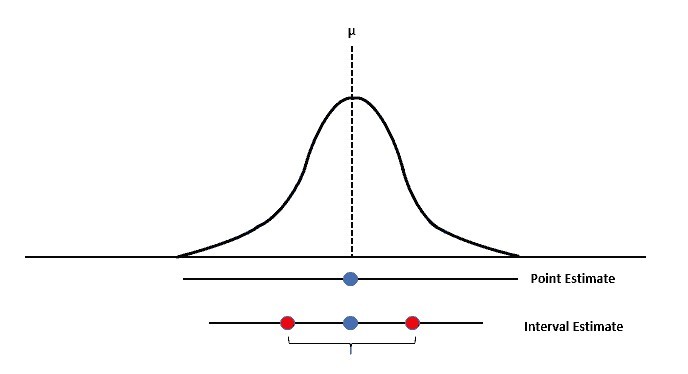
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Interval Estimation

Interval Estimation of the Population Mean Interval Estimation of the Population Proportion

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Interval Estimation

* Point estimators are not perfect!
  + They do not provide the exact value of the population parameter
* An **interval estimate**
  + computed by adding and subtracting a value, called the **margin of error**, to the point estimate.
* The general form of an interval estimate is:

Point estimate  Margin of error

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Interval Estimation

**Interval Estimation of the Population Mean:**

* An interval estimate provides information about how close the point estimate is to the value of the population parameter.
* General form of an interval estimate of a population mean is:

*x*  Margin of error

* General form of an interval estimate of a population proportion is:

*p*  Margin of error

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Interval Estimation

**Interval Estimation of the Population Mean (cont.):**

For any normally distributed random variable:

* 90% of the values lie within 1.645 standard deviations of the mean.
* 95% of the values lie within 1.960 standard deviations of the mean.
* 99% of the values lie within 2.576 standard deviations of the mean.

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Interval Estimation

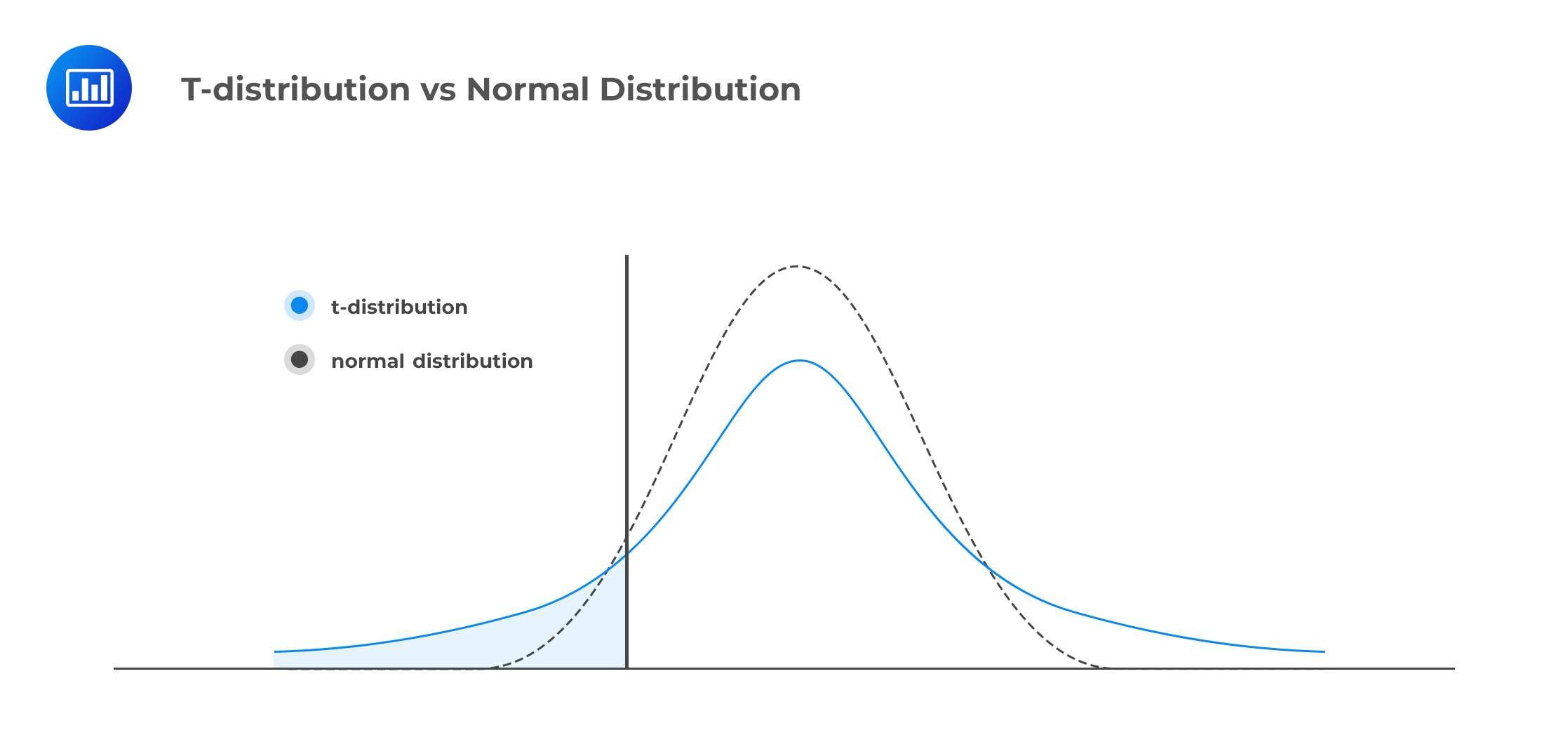
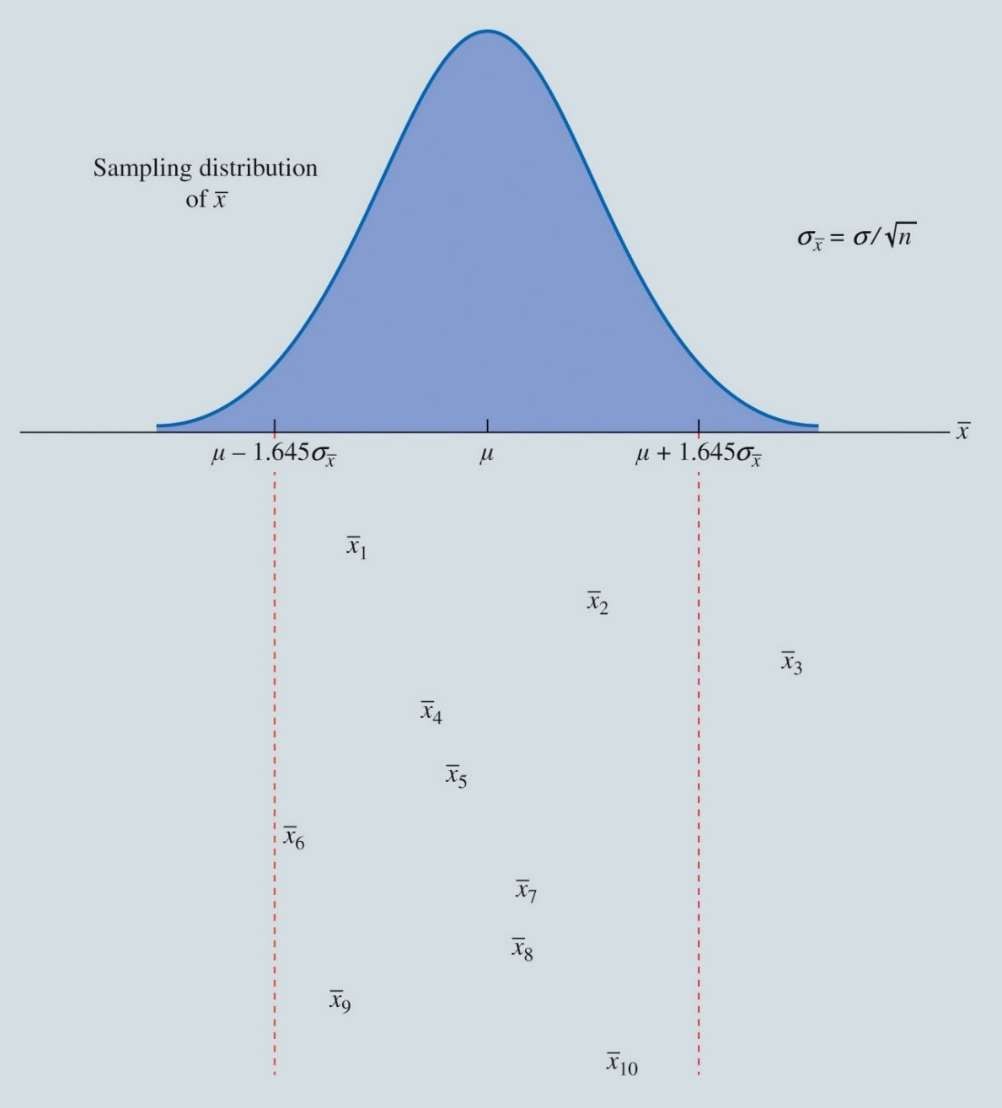
Sampling Distribution of the Sample Mean

How many of the 𝑥'𝑠 are in between the lines?

What does this mean?

* Remember, we do not generally know the population standard deviation, 𝜎
  + We have to use the sample data to estimate:
    - 𝜎 and 𝜇
  + This introduces more uncertainty about the distribution values of 𝑥.

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Interval Estimation

* To address this additional source of uncertainty
  + Use a probability distribution known as the *t* **distribution**:
    - A family of similar probability distributions.
    - The shape of each depends on a the **degrees of freedom**.
    - Similar in shape to the **standard normal distribution**, but wider.

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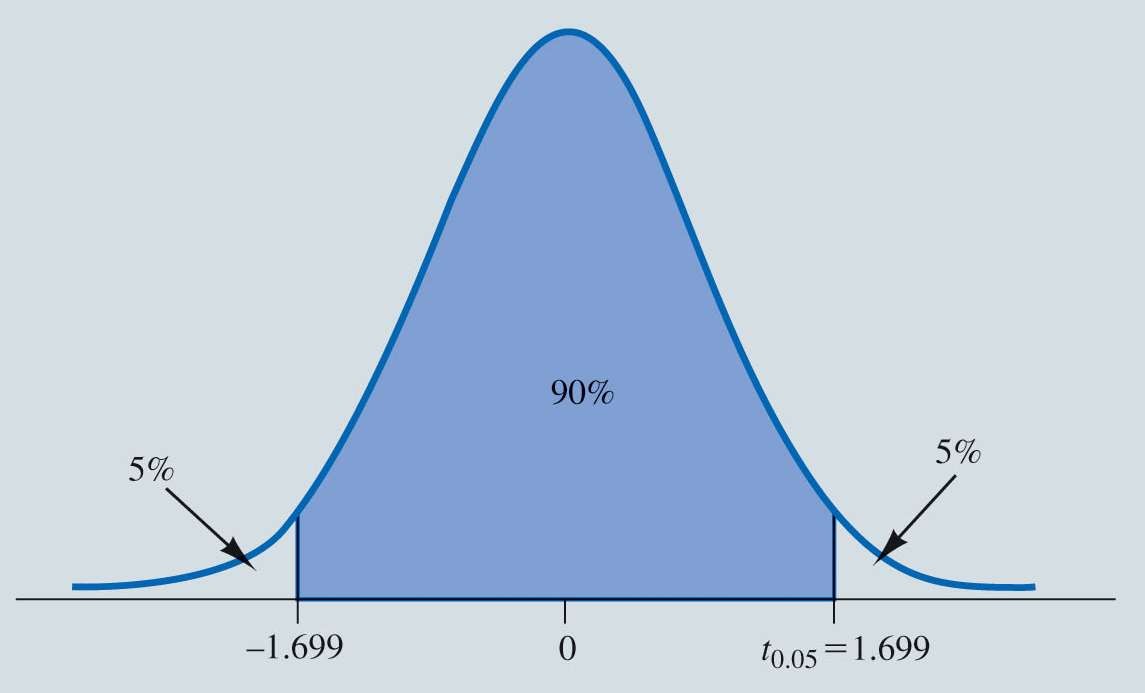
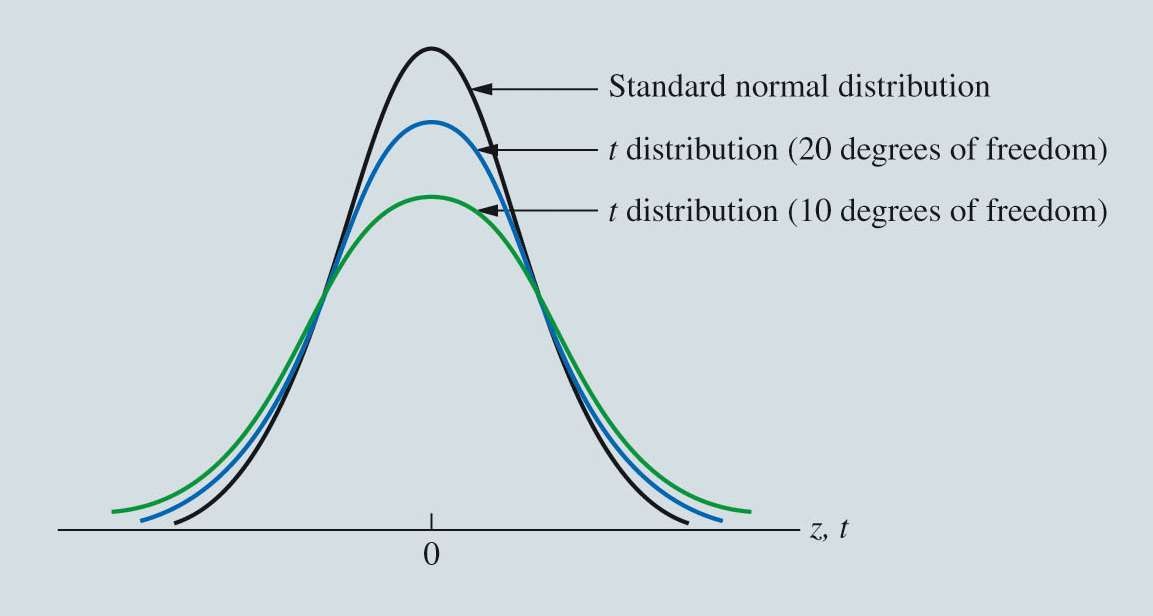


Interval Estimation

Comparison of the Standard Normal Distribution with *t* Distributions with 10 and 20 Degrees of Freedom

As the degrees of freedom increase, the *t* distribution narrows, its peak becomes higher, and it becomes more similar to the standard normal distribution.

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Interval Estimation

*t* Distribution with 29 Degrees of Freedom

**Use Excel′s T.INV.2T function to find the value from a** 𝒕 **distribution such that 95% of the distribution is included in the interval** ± 𝒕 **for 29 degrees of freedom.\***

45

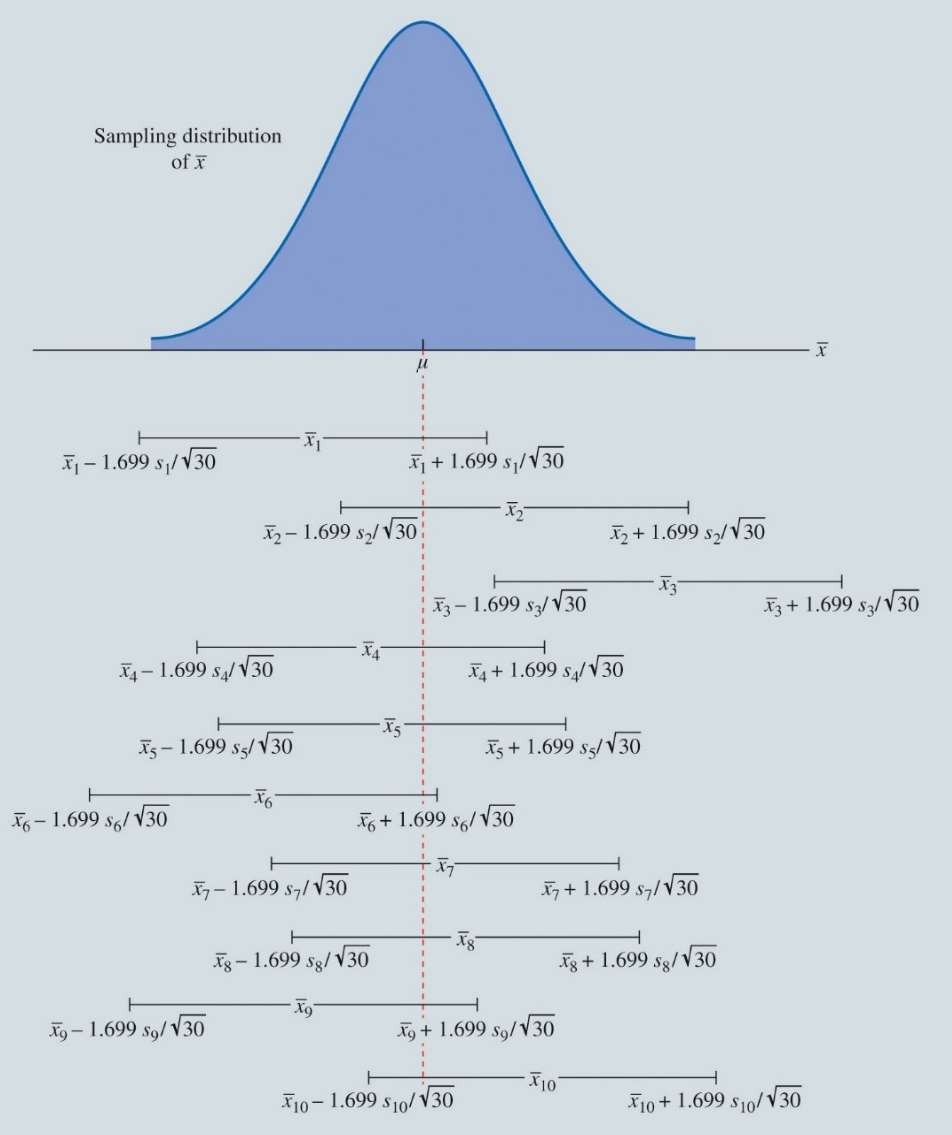


Interval Estimation

Intervals Formed Around Sample Means from 10 Independent Random Samples

* Approximately 90% of all the intervals constructed will contain the population mean
* We are approximately 90% confident that the interval will include the population mean:
  + The value of 0.90 is referred to as the **confidence coefficient**.
  + The interval is called the 90% **confidence interval**.

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Interval Estimation

* The **level of significance**
  + is the probability that the interval estimation procedure will generate an interval that does not contain the population mean:

** = level of significance = 1  confidence coefficient

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